EX 7 :

$f(x)=\dfrac{x+2}{(x+1)^4}$

$f(x)=\dfrac{a}{x+1}+\dfrac{b}{(x+1)^2}+\dfrac{c}{(x+1)^3}+\dfrac{d}{(x+1)^4}$

1. *a)* $\lim\limits_{x\to(-1)^+}(x+1)^4.f(x)=$

$\lim\limits_{x\to(-1)^+}(x+2)=1$

$\lim\limits_{x\to+\infty}(x+1).f(x)=$

$\lim\limits_{x\to+\infty}\dfrac{(x+1)(x+2)}{(x+1)^4}=\lim\limits_{+\infty}\dfrac{x^2}{x^4}$

$=\lim\limits_{+\infty}\dfrac{1}{x^2}=0$

b) $(x+1)^4.f(x)=$

$a(x+1)^3+b(x+1)^2+c(x+1)+d$

$\lim\limits_{x\to(-1)^+}(x+1)^2 f(x)=d$

or $\lim\limits_{x\to(-1)^+}(x+1)^4 f(x)=1$ *d'où* $d=1$

$\lim\limits_{x\to+\infty}(x+1)f(x)=$

$\lim\limits_{x\to+\infty}(a+\dfrac{b}{x+1}+\dfrac{c}{(x+1)^2}+\dfrac{d}{(x+1)^3})=a$

$\lim\limits_{x\to+\infty}(x+1)f(x)=0$

d'où $a=0$

$f(x)=\dfrac{b}{(x+1)^2}+\dfrac{c}{(x+1)^3}+\dfrac{1}{(x+1)^4}$

2. *a)* $\dfrac{x+2}{(x+1)^4}=\dfrac{b}{(x+1)^2}+\dfrac{c}{(x+1)^3}+\dfrac{1}{(x+1)^4}$

$\Rightarrow\dfrac{x+2}{(x+1)^4}-\dfrac{1}{(x+1)^4}=\dfrac{b}{(x+1)^2}+\dfrac{c}{(x+1)^3}$

$\Rightarrow\dfrac{1}{(x+1)^3}=\dfrac{b}{(x+1)^2}+\dfrac{c}{(x+1)^3}$

$\Rightarrow\dfrac{1-c}{(x+1)^3}=\dfrac{b}{(x+1)^2}$

b) $\dfrac{1-c}{(x+1)^3}=\dfrac{bx+b}{(x+1)^3}$

$\Rightarrow\begin{cases}b=0\\b=1-c\end{cases}\Rightarrow\begin{cases}b=0\\c=1\end{cases}$

d'où

$f(x)=\dfrac{1}{(x+1)^3}+\dfrac{1}{(x+1)^4}$

3. $F(x)=\dfrac{-1}{2(x+1)^2}-\dfrac{1}{3(x+1)^3}+k$

$F(0)=2\Rightarrow-\dfrac{1}{2}-\dfrac{1}{3}+k=0\Rightarrow k=\dfrac{5}{6}$

d'où

$F(x)=\dfrac{5}{6}-\dfrac{1}{2(x+1)^2}-\dfrac{1}{3(x+1)^3}$

EX 8 :

$f(x)=\dfrac{2x+1}{(x-2)^3}\quad,x\in\mathbb{R}\setminus\{2\}$

1. $f(x)=\dfrac{2(x-2)+5}{(x-2)^3}$

$f(x)=\dfrac{2}{(x-2)^2}+\dfrac{5}{(x-2)^3}$

2. $F(x)=\dfrac{-2}{x-2}-\dfrac{5}{2(x-2)^2}$

$F(x)=\dfrac{-4x+3}{2(x-2)^2}$

EX 9 :

$f(x)=\dfrac{1}{\sqrt{1-x^2}}\,;x\in\,]-1,1[$

F est dérivable sur $]-1,1[$

$\begin{cases}F'(x)=f(x)\\F(0)=0\end{cases}$

$g(x)=F(\sin x)$

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